**POSTGRADUATE THIRD SEMESTER EXAMINATIONS, 2021**

**Subject: Mathematics** **Course ID: 32155**

**Course Code: Math-305ME (Old)** **Course Title: Dynamical System (Old)**

**Full Marks: 40** **Time: 2 Hours**

**<u>The figures in the margin indicate full marks</u>**

**Notations and symbols have their usual meaning**

Answer any five questions. 8x5=40

1. (a) Write down the conditions for existence of unique solution of the following system of differential equations

$$\frac{dx}{dt} = a_{11}(t)x + a_{12}(t)y + F_1(t)$$
$$\frac{dy}{dt} = a_{21}(t)x + a_{22}(t)y + F_2(t).$$

(b) Find the solution of the homogeneous linear system

$$\frac{dx_1}{dt} = 4x_1 - x_2$$
$$\frac{dx_2}{dt} = x_1 + 2x_2.$$

2+6

2. Consider the linear system

$$\frac{dx_1}{dt} = x_1 + 3x_2$$
$$\frac{dx_2}{dt} = 3x_1 + x_2.$$

Find the solution of the corresponding uncoupled system and sketch the phase portrait in both x-plane and y-plane. 8

3. (a) Define hyperbolic and non-hyperbolic equilibrium point.

(b) Consider the non-linear system

$$\frac{dx_1}{dt} = -2x_2 + x_2x_3 - x_1^3$$
$$\frac{dx_2}{dt} = x_1 - x_1x_3 - x_2^3$$
$$\frac{dx_3}{dt} = x_1x_2 - x_3^3.$$

Check whether the trivial equilibrium point is hyperbolic or non-hyperbolic. Also investigate the stability of the equilibrium point. 2+6

4. (a) State the Bendixson-Dulac'scriterion for non-existence of close orbit.

(b) Investigate the global stability of the equilibrium point of the following system of equations

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = rx - y - xz$$
$$\frac{dz}{dt} = xy - bz.$$

2+6

5. Consider the Van der Pol equation

$$\frac{d^2y}{dt^2} - \mu(1 - y^2)\frac{dy}{dt} + y = 0.$$

Transform the above second order equation as a first order system of equations. Determine the steady state(s) and their stability. Also discuss whether the system undergoes Hopf bifurcation for gradual changes in the parameter $\mu$.

8

6. (a) Consider the difference equation

$$x_{n+1} = f(x_n).$$

Discuss about the local stability and asymptotic stability of a fixed point of the above difference equation.

(b) Consider the logistic difference equation

$x_{n+1} = \mu x_n(1 - x_n)$ when $\mu > 0$.

Find the fixed points and investigate their stability. 4+4

7. (a) Find the solution of the difference equation

$$x_{n+1} = 2x_n + 5^n, \quad x_1 = \frac{1}{2}.$$

(b) Solve the following system of difference equations

$x_{n+1} = 6x_n - 3y_n$

$y_{n+1} = 2x_n + y_n.$

Also check the stability of the solution. 3+5

8. (a) Consider the non-linear difference equation

$x_{n+1} = x_n^2 + 3x_n.$

Find the equilibrium points and determine their stability.

(b) Consider the system of non-linear difference equations

$x_{n+1} = 2x_n - y_n^2$

$y_{n+1} = y_n - 5x_n^2.$

Find the equilibrium points and determine their stability. 4+4